

The Lifetime of the Universe

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Abstract

Current observations of the fraction of dark energy and a lower limit on its tension, coupled with an assumption of the non-convexity of the dark energy potential, are used to derive a lower limit of 26 billion years for the future age of the universe. Conversely, our ordered observations, coupled with an assumption that observers are smaller than the universe, are used to argue for an upper limit of about $e^{10^{50}}$ years if the universe eventually undergoes power-law expansion, and an upper limit of only about 10^{60} years left for our universe if it continues to expand exponentially at the current rate.

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I. INTRODUCTION

A long-standing question in cosmology is the future lifetime of the universe. With traditional Friedman-Robertson-Walker dust models, it was thought that the universe would last forever if the deceleration parameter were $q_0 \leq 1/2$ and would recollapse in finite time if $q_0 > 1/2$. However, these dust models have been consigned to the dustbin, and the question has become quintessentially more difficult.

Here I shall give simple arguments for getting both lower and upper limits on the future lifetime of the universe.

II. LOWER LIMITS ON THE LIFETIME OF THE UNIVERSE

The recent evidence for cosmic acceleration [1, 2] is often interpreted as that for a positive cosmological constant and eternal expansion of the universe (e.g., in early WMAP press releases [3]).

However, many people have emphasized that the dark energy giving the current acceleration could be a scalar field ϕ with a potential $V(\phi)$ that could go negative and lead to a collapsing universe. See, for example, [4, 5, 6, 7, 8, 9]. Here I shall summarize a simplified form of the argument and get a w -dependent lower limit on the lifetime if $V(\phi)$ is not convex.

Scholars have dusted off the dust model by adding a scalar field $\phi(t)$ with potential $V(\phi)$. With $8\pi G/3 = c = 1$, $\dot{f} \equiv df/dt$, $f' \equiv df/d\phi$, $\dot{\phi} \equiv -v$, and $K \equiv \frac{1}{2}\dot{\phi}^2 \equiv \frac{1}{2}v^2$, a spatially-flat ($k = 0$) FRW scalar-dust universe obeys

$$\begin{aligned} H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 = \rho_m + \rho_s = \frac{\text{const.}}{a^3} + \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho_m + K + V, \\ \ddot{\phi} + 3H\dot{\phi} + V' &= 0, \end{aligned} \tag{1}$$

where $\rho_m = \text{const.}/a^3$ is the energy density of the dust, and $\rho_s = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ is the energy density of the scalar field, which has pressure $P_s = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.

These equations imply

$$\begin{aligned} \dot{\rho}_m &= -3H\rho_m, \\ \dot{\rho}_s &= -3H(\rho_s + P_s) = -3H(2K), \end{aligned} \tag{2}$$

which further imply

$$\dot{H} = -\frac{3}{2}(\rho + P) = -\frac{3}{2}(\rho_m + 2K) = -\frac{3}{2}(H^2 + K - V) < 0. \quad (3)$$

Also, $\dot{v} \equiv -\ddot{\phi} = V' - 3Hv$, so if V' stays larger than $3Hv$ as ϕ decreases, v and $K \equiv \frac{1}{2}v^2$ will continue to increase, keeping $-\dot{H}$ bounded away from zero. Then H will decrease indefinitely to $-\infty$, and the universe will end in a Big Crunch.

Eliminate the dust density by using $\dot{\rho}_m = -3H\rho_m$. Then one has three coupled first-order ordinary differential equations, $\dot{\phi} = -v$, $\dot{v} = V' - 3Hv$, and $\dot{H} = -\frac{3}{2}(H^2 + \frac{1}{2}v^2 - V)$. The time does not appear explicitly, so one can eliminate it and use $f' \equiv df/d\phi = \dot{f}/\dot{\phi} = -\dot{f}/v$ to get two coupled first-order ordinary differential equations:

$$\begin{aligned} v' &= 3H - \frac{V'}{v}, \\ H' &= \frac{3(H^2 - V)}{2v} + \frac{3}{4}v. \end{aligned} \quad (4)$$

Or, we can use the first of these two equations to eliminate H and get one second-order ordinary differential equation, simpler when written in terms of $K \equiv \frac{1}{2}v^2$ rather than v :

$$K'' = \frac{(K' + V')(3K' + V')}{4K} + \frac{9}{2}K - 9V - V''. \quad (5)$$

In principle, we need two boundary conditions, e.g., v and H (or K and K') at some ϕ , or ϕ and K when the dimensionless density parameter Ω_m reaches its present value Ω_{m0} , $\Omega_m = (H^2 - K - V)/H^2 = \Omega_{m0}$. However, generic data at $\Omega_m = \Omega_{m0}$ will evolve back to $K = \infty$ at $H = \infty$ (at the Big Bang, $a = 0$). Boundary conditions arising out of early-universe inflation would suggest restricting to finite K at $H = +\infty$, which leaves only one arbitrary parameter, say the initial scalar field value, $\phi = \phi_i$, at the beginning of the universe, at $H = +\infty$.

If $V(\phi) \approx V_i - V'_i(\phi_i - \phi) + \frac{1}{2}V''_i(\phi_i - \phi)^2$ for $0 < \phi_i - \phi \ll 1$, then Eq. (5) is solved with

$$K \approx \frac{1}{3}V'_i(\phi_i - \phi) - \frac{3}{10}(6V_i + V''_i)(\phi_i - \phi)^2 \quad (6)$$

for $0 < \phi_i - \phi \ll 1$. This gives

$$t = \int_{\phi_i}^{\phi} \frac{-d\tilde{\phi}}{\sqrt{2K(\tilde{\phi})}} \approx \sqrt{\frac{6(\phi_i - \phi)}{V'_i}}, \quad (7)$$

implying $\phi \approx \phi_i - \frac{1}{6}V'_i t^2$ and

$$H = \frac{K' + V'}{\sqrt{18K}} \approx \sqrt{\frac{2V'_i}{27(\phi_i - \phi)}} \approx \frac{2}{3t}, \quad (8)$$

which describes the initial dust-dominated phase.

Then for a chosen initial value ϕ_i , evolve $\phi_i - \phi$ to the point where $\Omega_m = 1 - (K + V)/H^2 = 1 - 18K(K + V)/(K' + V')^2 = \Omega_{m0}$, and there evaluate

$$w \equiv \frac{P_s}{\rho_s} = \frac{K - V}{K + V} = w(\phi_i, \Omega_{m0}). \quad (9)$$

Thus for a given scalar field potential function $V(\phi)$ and given present values Ω_{m0} and w_0 , solving $w(\phi_i, \Omega_{m0}) = w_0$ for ϕ_i gives $\phi_i = \phi_i(\Omega_{m0}, w_0)$ and hence a unique model. Then one can solve for the ratio of the future time until the Big Crunch to the past time since the Big Bang,

$$R = R(\Omega_{m0}, w_0) = \frac{t_c}{t_0} = \frac{\text{future time to crunch}}{\text{past time since bang}}. \quad (10)$$

If $V(\phi)$ is convex with a nearby cliff, in principle the universe could fall off the cliff and recollapse in the next minute. However, with the hope that you will be able to read the rest of this paper, I shall assume that this is implausible.

Therefore, for concreteness, assume that $V(\phi)$ is not convex. Then the smallest $R(\Omega_{m0}, w_0) = t_c/t_0$ would be for the case of a linear potential, $V(\phi) = \beta\phi$.

With $8\pi G/3 = c = 1$, the dimensions of mass, length and time are the same, the scalar field ϕ is dimensionless, and β has the dimensions of V , which is that of energy per length cubed, or of mass per length per time squared, or of inverse time squared. Therefore, we may temporarily choose units of time so that $\beta = 1$. (This gives $\hbar \neq 1$, but that is irrelevant for our classical calculation.) Then $V(\phi) = \phi$, $V' = 1$, and $V'' = 0$, so with $x = \phi_i - \phi$,

$$\frac{d^2 K}{dx^2} = \frac{1}{4K} \left(1 - \frac{dK}{dx}\right) \left(1 - 3\frac{dK}{dx}\right) + \frac{9}{2}K - 9\phi_i + 9x, \quad (11)$$

with the boundary condition that $K = \frac{1}{3}x - \frac{9}{5}\phi_i x^2 + O(x^3)$ for $x \ll 1$. Here ϕ_i is the single free parameter, which can be varied to get any desired value for w_0 when $\Omega_m = \Omega_{m0}$.

When one evolves Eq. (11) numerically, $K(x) \equiv \frac{1}{2}v^2 \equiv \frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(\rho_s + P_s)$ increases monotonically with x , and $H \equiv \dot{a}/a = (1 - dK/dx)/\sqrt{18K}$ decreases monotonically from $+\infty$ at the dust-dominated Big Bang to $-\infty$ at the stiff-scalar-dominated Big Crunch.

Take the past age to be $t_0 = 10^8 \text{yr}/\alpha = 13.7 \text{ Gyr}$ and $\Omega_{m0} = 0.27$ [10]. Then for a non-convex scalar field potential, the lower limit on the future lifetime t_c (until the Big Crunch) is a function of $w_0 = P_s/\rho_s = (\text{scalar field pressure})/(\text{scalar field energy density})$, with values given in the following table:

w_0	$R = \frac{t_c}{t_0}$	$\frac{0.128}{1+w_0} - 1.68 - 3.81w_0$	t_c in Gyr
-0.78	1.90	1.87	26
-0.86	2.44	2.51	33
-0.90	2.99	3.03	41
-0.95	4.68	4.50	64
-0.98	9.21	8.45	130
-0.99	16.3	14.9	220
-0.999	135	130	1 800
-0.9999	1 300	1 300	18 000
-0.99999	12 900	12 800	180 000
-0.999999	128 000	128 000	1 800 000

The formula at the top of the third column is a crude fit to $R(\Omega_{m0}, w_0)$ for $\Omega_{m0} = 0.27$; one can see from the table of values how accurate it is.

Therefore, if we assume that $V(\phi)$ is non-convex and take $t_0 = 13.7 \text{ Gyr}$, $\Omega_{m0} = 0.27$, and $w_0 \leq -0.78$ [10], we find a lower limit of the future lifetime of the universe of 26 Gyr, or a minimum total lifetime of the universe of 40 Gyr.

If we had not only w_0 but also $(H\dot{w})_0$, the present value of $H\dot{w}$, we could get lower limits on lifetimes with nonlinear potentials $V(\phi)$ with some lower limits on, say, $V'''(\phi)$.

III. UPPER LIMITS ON THE LIFETIME OF THE UNIVERSE

Can we also get a finite upper limit on the lifetime of the universe? Apparently not from classical arguments and current observations, which are consistent with a true cosmological constant $\Lambda > 0$, $w_0 = -1$, and asymptotically de Sitter expansion forever.

But several theorists have argued that de Sitter space is actually unstable [4, 11, 12, 13, 14, 15, 16]. The lifetime is argued to be

$$\tau < \text{recurrence time} \sim (\text{number of states}) \sim e^S = e^{A/4} = e^{3\pi/\Lambda}$$

$$\begin{aligned}
&= \exp\left(\frac{\pi}{\Omega_\Lambda H^2}\right) = \exp\left(\frac{1.03 \times 10^{122}}{\Omega_\Lambda h_0^2}\right) = \exp\left[2.80 \times 10^{122} \left(\frac{0.73}{\Omega_{\Lambda_0}}\right) \left(\frac{0.71}{h_0}\right)^2\right] \\
&\approx e^{e^{e^{e^{e^{54823}}}}} \approx 10^{10^{10^{10^{31945}}}} \gg \gg \text{googleplex} = 10^{10^{10^2}} \approx e^{e^{e^{e^{e^{52727}}}}} \approx 10^{10^{10^{10^{30103}}}}. \quad (12)
\end{aligned}$$

Here I shall develop a cosmological doomsday argument that we have observational evidence for a lifetime \llll googleplex.

Assume an observation is described by a localized positive operator \hat{A} , such as a projection operator onto some state of a brain with suitable environment to permit the observation. Then for a universe larger than the size of the localized operator, $\langle \psi | \hat{A} | \psi \rangle > 0$ for virtually any state $|\psi\rangle$, so the observation will have some positive probability of occurring (e.g., as a vacuum fluctuation). The only way out that I see would be if observers and observations require the entire size of the universe, such as a projection operator onto some total value of a globally conserved quantity.

Assume a human observer has finite size and just requires a 1 kg brain to last 0.1 second. The dimensionless action for this is

$$I(\text{brain}) \sim \frac{E\Delta t}{\hbar} \sim \frac{(1 \text{ kg})(3 \times 10^8 \text{ m/s})^2(0.1 \text{ s})}{10^{-34} \text{ J} \cdot \text{s}} \sim 10^{50}. \quad (13)$$

Therefore, in any spacetime volume $V_4(\text{brain}) \sim (10 \text{ cm})^3(0.1 \text{ s}) \sim 10^{144} l_{\text{Pl}}^4 \sim e^{331} l_{\text{Pl}}^4$, one would expect a human observation to have probability $P \geq P(\text{brain}) \sim e^{-I} \sim e^{-10^{50}}$. Thus observations should occur at a rate per Planck 4-volume of

$$R > \frac{P(\text{brain})}{V_4(\text{brain})} \sim e^{-331-10^{50}} \sim e^{-10^{50}}. \quad (14)$$

Assuming $N \sim (10^{11} \text{ people})(10 \text{ observations/s})(10^9 \text{ s}) \sim 10^{21} \sim e^{48}$ observations during past human history, we just need a spacetime volume of

$$V_4 \sim NR^{-1} < e^{48+331+10^{50}} \sim e^{10^{50}} \quad (15)$$

to get more observations by vacuum fluctuations than have occurred during past human history.

But if our spacetime lasts long enough to give a 4-volume (of our comoving part) $V_4 \gg e^{I(\text{brain})} \sim e^{10^{50}}$, then almost all human observations would be from vacuum fluctuations. However, these would almost entirely be much more disordered than what we experience. Therefore, our ordered observations would be highly atypical in this scenario.

This extreme atypicality is like an extremely low likelihood, counting as very strong observational evidence against any theory predicting such a long-lived universe with a quantum state that can allow localized observations.

Thus I predict that the universe will not last long enough to give 4-volume $> e^{10^{50}}$.

One gets various upper limits on the future lifetime from different assumptions about the future expansion. If all of the energy density of the universe, including the dark energy, decays to radiation with $\rho \sim 1/a^4$, then a $k = 0$ FRW universe will asymptotically have $a \sim t^{1/2}$ and $V_4 \propto \int a^3 dt \sim t^{5/2}$, which would exceed $e^{I(\text{brain})} \sim e^{10^{50}}$ at $t \sim \left(e^{10^{50}}\right)^{2/5} = e^{0.4 \times 10^{50}} \sim e^{10^{49.6}}$. If one has instead a $k = -1$ FRW universe, asymptotically $a \sim t$ and $V_4 \sim t^4$, exceeding $e^{I(\text{brain})}$ at $t \sim \left(e^{10^{50}}\right)^{1/4} = e^{0.25 \times 10^{50}} \sim e^{10^{49.4}}$. Thus for any power-law expansion with exponent of order unity, I would predict that the universe will not last past

$$t \approx e^{10^{50}} \sim e^{e^{115}} \sim e^{e^{e^{e^{e^{.443}}}}} \llll \text{googlexplex} \approx e^{e^{e^{e^{e^{.52727}}}}} . \quad (16)$$

On the other hand, if the dark energy has a positive minimum energy density ($\Lambda > 0$), then the universe asymptotically has $a \sim e^{H_\Lambda t} = e^{\sqrt{\Lambda/3}t}$, so $V_4 \propto \int a^3 dt \sim \int e^{3H_\Lambda t} dt \sim e^{3H_\Lambda t}$, which would exceed $e^{I(\text{brain})} \sim e^{10^{50}}$ at $t \sim I/(3H_\Lambda) \sim 10^{50}/(3H_\Lambda) \sim 10^{50}/\sqrt{3\Lambda}$. For $H_\Lambda = \sqrt{\Lambda/3} = \sqrt{\Omega_{\Lambda_0} H_0^2} = \sqrt{\Omega_{\Lambda_0}} h_0 / (9.77813 \text{ Gyr}) \sim 1.06 \times 10^{-61} t_{\text{Pl}}^{-1}$, one gets $t \sim 10^{50} (5.4 \times 10^9 \text{ yr}) \sim 10^{60} \text{ yr} \sim 10^{111} t_{\text{Pl}}$.

Therefore, if the current dark energy were due to a true cosmological constant, it would produce too much spacetime volume to be consistent with our non-vacuum-fluctuation (i.e., ordered) observations after only about 10^{60} years.

What are the implications for the string landscape or stringscape? The observational evidence for $V_4 < e^{10^{50}}$ seems to imply that negligibly few universes permitting observers would arise from positive metastable minima of the stringscape potential with tunneling lifetime greater than about $10^{50} t_0 \sim 10^{111} t_{\text{Pl}}$.

Is it possible that all life-permitting positive metastable minima decay away within $10^{50} t_0 \sim e^{254} t_{\text{Pl}}$? Or, if longer-living ones exist, can they possibly be overwhelmed by non-extreme regions that are nevertheless low and flat enough for observers? Or might it be true that the stringscape simply has *no* positive local minima at all?

For either of the latter two possibilities, we might speculate that by measurements of $w(t) > -1$, we could observe our universe already sliding toward oblivion.

IV. CONCLUSIONS

Thus I would predict, based on current observations and some plausible assumptions (non-convex $V(\phi)$ and observers smaller than the universe), that the future lifetime of our universe (at least in a form permitting observers) has both lower and upper bounds:

$$26 \text{ Gyr} < t_{\text{future}} < e^{10^{50}} \text{ Gyr or } 10^{51} \text{ Gyr}, \quad (17)$$

with the first possibility on the right being for future power-law expansion, and the second being for exponential expansion at the rate given by the current dark energy.

Our observations also suggest that the dark energy of our universe is not near a positive local minimum ($\Lambda > 0$), unless it can decay within about $10^{60} \text{ yr} \sim 10^{111} t_{\text{Pl}} \sim e^{254} t_{\text{Pl}}$. Furthermore, the string landscape or stringscape should not have *any* significant long-lived positive metastable minima with false vacua suitable for observers (e.g., with lifetime longer than about 10^{60} yr for a minimum with roughly the same energy density as the observed dark energy), for such minima would expand to give a huge volume and too many observers from vacuum fluctuations to be consistent with our ordered observations being typical.

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